Measuring the Stringency of Land Use Regulation: A Shadow Price Approach

Junfu Zhang*

Abstract

I propose a shadow price approach to measuring the stringency of land use regulation. A regulation is considered more restrictive if the land developer is willing to pay a larger amount for a marginal relaxation of the regulation. I show that existing theory-based measures of land use stringency are either equivalents or variations of this shadow price measure. Using data from China, I demonstrate that it is possible to compare the stringency of two kinds of land use regulation, a key advantage of this shadow price approach.

Keywords: Land use, regulation, stringency, shadow price.

JEL Classification: R14, R52.

[Accepted for publication at JUE: Insights]

1 Introduction

Many studies have used quantitative measures of land use stringency. Some aim to understand what factors determine the restrictiveness of land use regulations; others examine the effects of land use regulations on various outcomes such as local housing prices. Most of the stringency measures used in the literature are pragmatically constructed atheoretical indexes. For example, Levine (1999) counts the number of enacted local growth controls in California, and Gyourko et al. (2008) calculate a residential land use regulatory index for U.S. states and metropolitan areas.[1] These measures do not have a theoretical explanation. More importantly, they do not properly take into account local conditions. Whereas a ten-story height limit can be very

*Professor of Economics, Clark University, 950 Main Street, Worcester MA 01610; Tel: 508-793-7247; E-mail: juzhang@clarku.edu. This paper has benefited from comments by Jan Brueckner, Shihe Fu, Edward Glaeser (coeditor), and two anonymous referees. The usual disclaimer applies.

[1]Many other studies have used the same or similar index measures. See, for example, Pollakowski and Wachter (1990), Malpezzi (1996), Mayer and Somerville (2000), Quigley and Rosenthal (2005), Ihlanfeldt (2007), Glaeser and Ward (2009), Huang and Tang (2012), Kok et al. (2014), Turner et al. (2014), Albouy and Ehrlich (2018), and Gyourko et al. (2021).

restrictive in Manhattan of New York City, it is unlikely to be binding in Manhattan, Kansas. Thus one would like a stringency measure to be defined relative to local conditions.

Three theoretically motivated measures of land use stringency have this desired property. The first is given by Fu and Somerville (2001). In a study of site density restrictions in China, they argue that "the marginal land value of development density … describes the extent to which density restrictions constrain development at any given site." That is, the restriction is more stringent if a marginal relaxation increases land value by a larger amount. I will refer to this as the *marginal value* measure of stringency. The second is proposed by Glaeser et al. (2005). Investigating the rising housing prices in New York City, Glaeser and coauthors define the regulatory tax as the difference between the market price of a housing unit and the marginal cost of that unit (if without regulations). They argue that a larger difference represents more restrictive regulations. Following their terminology, I will call this the *regulatory tax* measure.[2] The third measure is proposed by Brueckner et al. (2017). They argue that the stringency of a land use regulation should be measured as the ratio of the profit maximizing choice (in absence of the regulation) to the regulated level. I will refer to this ratio as the *relative gap* measure of stringency because it shows the relative difference between the developer's optimal choice and the government's regulated level. Since this ratio is unobservable, Brueckner and coauthors propose to estimate the elasticity of land price to the regulated level of constraint, which they prove to be positively correlated with the relative gap measure of stringency.[3] Tan et al. (2020) show that this relative gap measure is positively correlated with the share of land cost in housing value, thus they propose to measure land use stringency using estimated land share.

In this short paper, I propose an alternative theoretically attractive approach to measuring the stringency of land use regulation. Assume that the land developer maximizes profit subject to the constraint of a regulation. By the envelope theorem, the Lagrangian multiplier of this constrained optimization problem can be interpreted as the amount of profit loss resulting from a marginal increase in the tightness of the regulation. I argue that this Lagrangian multiplier can be used as a measure of land use stringency. By this measure, a regulation is more stringent if tightening it slightly leads to a larger profit loss for the developer. In other words, a regulation is more restrictive if the land developer has to pay a higher shadow price.

Although seemingly unrelated, the existing theory-based measures of stringency are either equivalents or variations of the shadow price measure. Under perfect competition among developers, the change in developer's profit is fully reflected in the change in land price. Consequently, the marginal value measure is the same as the shadow price measure. The regulatory tax is exactly the extra profit a developer could make had regulations been relaxed to allow for the building of another unit of housing, thus it is essentially a shadow price paid by developers. As will be shown below, the elasticity estimated by Brueckner et al. (2017) and the land share

[2]This measure has since been adopted by some follow-up studies (e.g., Cheshire and Hilber 2008, Cheung et al. 2009, Sunding and Swoboda 2010, Glaeser and Gyourko 2018). Gyourko and Krimmel (2021) adapt this measure to estimate a "zoning tax" as the gap between extensive and intensive margin values of land.

[3]This measure is also adopted by other studies (e.g., Moon 2019, Brueckner and Singh 2020).

estimated by Tan et al. (2020) are both monotonic transformations of the shadow price measure. My formulation therefore provides a unified framework for understanding a range of recent empirical studies on land use stringency.

This shadow price measure of land use stringency is appealing in several aspects. Most importantly, it can be extended to measure and compare the stringency of multiple regulations. This is feasible simply because the Lagrangian method is capable of dealing with optimization under multiple constraints. In my empirical exercise, using data from China, I measure and compare the stringency of two regulations on residential development, a maximum floor area ratio (FAR) restriction and a minimum green coverage ratio (GCR) requirement.

2 Measuring Land Use Stringency as a Shadow Price: A Formal Argument

Assume that each developer has access to a constant returns to scale building technology given by the production function $H(K, L)$, where K units of capital and L units of land are combined to produce H units of housing. For simplicity, I ignore labor as a factor of production; one may assume that the price of capital includes labor cost needed for construction. Define $h \equiv \frac{H(K,L)}{L} = H\left(\frac{K}{L}, 1\right)$ as the quantity of housing produced on one unit of land (i.e., the floor area ratio) and $k \equiv \frac{K}{L}$ the amount of capital used on one unit of land (often referred to as "structural density"). The housing production function can be rewritten as $h(k)$, with $h' > 0$ and $h'' < 0$.

Markets are perfectly competitive, so a developer takes housing price (p), price of capital (i), and land rent (r) all as given. A developer chooses the optimal k to maximize the *pre-rent profit* from one unit of land (in order to compete for land with other developers). Given land price, this is equivalent to maximizing the overall profit. Also, the developer faces the constraint of a government regulation: $R(k) \leq \bar{R}$, where the government sets an upper limit on a certain aspect of housing production. $R(k)$ may be the building height or floor area ratio. Note that the direction of the inequality is imposed without loss of generality because $R(k)$ can be defined as the negative of a quantity for which the government has a minimum requirement.

The developer's constrained optimization problem is as follows:

$$\begin{gathered} \pi(p, i, \bar{R}) = \max_k ph(k) - ik \\ \text{subject to } R(k) \leq \bar{R}. \end{gathered} \tag{1}$$

The Lagrangian function of the developer's optimization problem is:

$$\mathfrak{L}(k, \lambda) = ph(k) - ik - \lambda \left[R(k) - \bar{R}\right]. \tag{2}$$

Kuhn-Tucker conditions require that $\lambda^* = 0$ when the constraint is not binding and $\lambda^* > 0$ when it is binding. By the envelope theorem, $\lambda^* = \frac{\partial \pi(p,i,\bar{R})}{\partial \bar{R}}$, which is the amount of profit loss when $\bar{R}$ decreases by one unit (or extra profit when $\bar{R}$ increases by one unit).

I argue that λ^* serves as an intuitive measure of the stringency of the land use regulation. The higher λ^* is, the larger the developer's extra profit will be when the policy constraint is relaxed marginally, and the more stringent the current regulation is. Since λ^* represents the "shadow price" the developer is willing to pay for a marginal relaxation of the policy constraint, I call it the shadow price measure of the stringency of the land use regulation.[4]

Glaeser et al. (2005, p. 336) propose the following measure of land use stringency: regulatory tax = market price of a housing unit - marginal cost of that unit (absent government barriers). They call it a tax because like a tax, regulatory constraints create this wedge between market price and marginal cost. To see the connection between this "regulatory tax" and my shadow price measure, notice that equation (1) can be rewitten as an optimization problem over h (instead of k). Let $k(h)$ be the inverse function of $h(k)$ and define $C(h) \equiv ik(h)$. We can rewrite equation (2) as

$$\mathfrak{L}(h, \lambda) = ph - C(h) - \lambda \left[R(k(h)) - \bar{R} \right]. \tag{2'}$$

Its first order condition requires

$$p - C'(h) = \lambda^* R'(k(h)) \cdot k'(h). \tag{3}$$

Since $R'(k(h)) \cdot k'(h)$ is independent of $\bar{R}$, the regulatory tax $(p - C'(h))$ and the shadow price measure (λ^*) are equivalent up to a multiplicative constant.[5] When the constraint in equation (1) is on the floor area ratio (i.e., $h \leq \bar{h}$), the two measures are exactly the same. Instead of using regression analysis, Glaeser et al. (2005) estimate marginal cost of housing units based on information from builders and then calculate regulatory tax by subtracting the estimated marginal cost from observed market price.

To make it easier to bring the shadow price measure to data, I invoke the zero profit condition under perfect competition among developers: $\pi(p, i, \bar{R}) = r$. It follows that $\lambda^* = \frac{\partial r}{\partial \bar{R}}$. That is, the stringency of the policy can be measured by the reaction of observed land prices to a marginal relaxation of the regulation. The policy is more restrictive if increasing the upper limit by one unit leads to a larger increase in land price. This is exactly the measure proposed by Fu and Somerville (2001).[6]

Brueckner et al. (2017) consider a specific constraint in equation (1): $h(k) \leq \bar{h}$. That is,

[4] van Soest et al. (2006) propose a shadow price approach to measure environmental stringency. Instead of looking at changes in profit, they define the shadow price as the increase in production costs. Furth (2021) is the only work I know of that interprets the Lagrangian multiplier in a developer's profit maximization problem as a possible measure of land use stringency.

[5] Unlike my formulation here, Glaeser et al. (2005) do not define the regulatory tax *for one unit of land*. Their implicit assumption is that the regulatory tax is the same for a wide range of land use levels.

[6] Fu and Somerville (2001, p.405) clearly recognize that their measure of land use stringency is related to "the increase in profit that would accrue to the landowner from a relaxation of the density restrictions." They do not directly quantify their measure of stringency. Instead, they regress log land price on log FAR together with a variety of locational attributes, and interpret the coefficients of locational attributes as their effects on land use stringency.

the government imposes an upper limit of the floor area ratio h, a policy commonly applied to residential development in China. (Note that this is equivalent to imposing an upper limit on the structural density k.) They propose to measure the stringency of this policy by $\frac{h^*}{\bar{h}}$, where $h^* = h(k^*)$ is the developer's optimal h given by the profit-maximizing structural density if there was no restriction. A higher $\frac{h^*}{\bar{h}}$ means that the developer would choose a relatively larger h if not restricted by $\bar{h}$, implying a more stringent regulation. Obviously $\frac{h^*}{\bar{h}}$ is not directly observable if $\bar{h}$ is binding. Brueckner and coauthors show that $\frac{h^*}{\bar{h}}$ is positively correlated with $\frac{\partial r/r}{\partial \bar{h}/\bar{h}} = \frac{\partial \ln r}{\partial \ln \bar{h}}$, the elasticity of land price to the regulated floor area ratio. They therefore proceed to estimate $\frac{\partial \ln r}{\partial \ln \bar{h}}$ from observed data and use it as an indicator of land use stringency. Note that in this case, the shadow price measure of stringency should be $\lambda^* = \frac{\partial r}{\partial \bar{h}}$. Thus Brueckner et al. (2017) estimated a rescaled version of it: $\frac{\partial \ln r}{\partial \ln \bar{h}} = \left(\frac{\bar{h}}{r}\right)\lambda^*$.[7] This easy conversion provides an alternative way to estimate the shadow price, especially when the log-log specification is preferred on statistical grounds.

Tan et al. (2020) examine the same specific constraint as Brueckner et al. (2017): $h(k) \leq \bar{h}$. They prove that the relative gap measure of stringency ($\frac{h^*}{\bar{h}}$) is positively correlated with the "ratio of per unit land price to housing value per land unit." They therefore study stringency empirically by estimating the land share. In my language here, Tan et al. (2020) estimate $\frac{r}{p\bar{h}}$, assuming the constraint is binding. Under perfect competition, $\frac{r}{p\bar{h}} = \frac{1}{p}\frac{\pi(p,i,\bar{h})}{\bar{h}}$, which is the average pre-rent profit per unit of housing (normalized by housing price). The shadow price measure $\frac{\partial \pi(p,i,\bar{h})}{\partial \bar{h}}$ is the marginal pre-rent profit. Since total profit is the integral of marginal profit, we have $\frac{1}{p}\frac{\pi(p,i,\bar{h})}{\bar{h}} = \frac{1}{p\bar{h}}\int_0^{\bar{h}} \frac{\partial \pi(p,i,\bar{t})}{\partial \bar{t}} d\bar{t}$.[8] Thus the empirical measure of Tan et al. (2020) is also a monotonic transformation of the shadow price measure. However, this transformation does not seem to have an easy justification.

In summary, among the three theory-based approaches to measuring land use stringency, the first one (Fu and Somerville 2001) is equivalent to my shadow price approach; the second one (Glaeser et al. 2005) is a special case of my shadow price approach, treating all relevant regulations together as a single constraint on supplied quantity of housing; the third one (Brueckner et al. 2017) is not directly observable and its empirical implementations are transformations of my shadow price measure. My formulation thus provides a unified framework for understanding these previously used measures of land use stringency. A few more remarks are in order:

First, this shadow price approach can be easily extended to cases with more than one regulatory constraint. This is feasible simply because the Lagrangian method allows for more than one constraint. Suppose the developer faces two constraints imposed by government regulations: $R_1(k) \leq \bar{R}_1$ and $R_2(k) \leq \bar{R}_2$. The associated Lagrangian is now:

$$\mathfrak{L}(k, \lambda_1, \lambda_2) = ph(k) - ik - \lambda_1 \left[R_1(k) - \bar{R}_1\right] - \lambda_2 \left[R_2(k) - \bar{R}_2\right]. \quad (4)$$

[7]Here $\frac{\bar{h}}{r}$ is not a multiplicative constant. If the constraint is binding, we have $r = p\bar{h} - ik(\bar{h})$ under perfect competition. It follows that $\frac{\bar{h}}{r} = \left[p - ik(\bar{h})/\bar{h}\right]^{-1}$, which is increasing in $\bar{h}$ (given the concave function $h(k)$).

[8]When $\bar{h}$ is binding, $\pi(p, i, \bar{h})$ is an increasing concave function of $\bar{h}$. Thus $\frac{\partial \pi}{\partial \bar{h}}$ and $\frac{\pi}{\bar{h}}$ are positively correlated and $\frac{\partial \pi}{\partial \bar{h}} < \frac{\pi}{\bar{h}}$ for all positive binding $\bar{h}$.

Then $\lambda_1^* = \frac{\partial \pi(p,i,\bar{R}_1,\bar{R}_2)}{\partial \bar{R}_1}$ and $\lambda_2^* = \frac{\partial \pi(p,i,\bar{R}_1,\bar{R}_2)}{\partial \bar{R}_2}$ serve as the stringency measures for the two regulations. Again, because zero profit implies $\lambda_1^* = \frac{\partial r}{\partial \bar{R}_1}$ and $\lambda_2^* = \frac{\partial r}{\partial \bar{R}_2}$, the two stringency measures can be estimated by regressing land price on changes in regulated levels. This is a key extension that is not explored by earlier studies.

Second, since the regulatory tax is a special case of the shadow price measure, my formulation here has made it easier to take the theoretical concept to data. Given the connection between developer's profit and land price, my formulation offers an alternative way to estimate the regulatory tax. In addition, under my approach, one does not have to treat all regulations together as a bundle (as in Glaeser et al. 2005); it is possible (at least in theory) to estimate the regulatory tax associated with any specific regulation. This last point will be further elaborated in the empirical section below.

Finally, compared to the relative gap measure of stringency proposed by Brueckner et al. (2017), this shadow price measure has clear advantages. The relative gap measure is an ordinal index. It allows us to check whether a regulation is more stringent in one location than another. However, the difference in this measure between the two locations is not easily interpretable. Also, it cannot be used to compare the stringency of two regulations. In contrast, the shadow price measure is a cardinal concept. If by the shadow price measure a regulation is twice as stringent in one location than another, it simply means that a marginal increase in the tightness of the regulation will cause a profit loss to developers that is twice as large in the first location. Similarly, by the shadow price measure one can compare the stringency of two different regulations in the same way. In addition, compared with the theoretical arguments given by Brueckner et al. (2017) and Tan et al. (2020), the logic behind this shadow price measure is simpler. (Indeed, it is embarrassingly simple.)

3 Comparing the Stringency of Two Types of Land Use Regulations: An Empirical Exercise

3.1 Data and specification

For empirical demonstration, I use land transaction data from cities in China, originally analyzed by Brueckner et al. (2017). The data come from the China Index Academy (CIA), the largest independent research institute in China specializing in real estate and land issues. One of the CIA's major products is its proprietary database on land transactions in over 200 cities across China. I use an extract of their data generated in early 2012. It contains information on over 30,000 residential land transactions during 2002-2011. More recent studies have used data collected by crawling a government website (see Fu et al. 2021). However, publicly available online data only contains information on one type of regulation: the floor area ratio (FAR) restriction. The CIA data is preferred for my purpose here because it also has information on whether a residential land parcel is subject to a minimum "green coverage ratio" (GCR) requirement. For example, a 35% GCR requirement means that the developer needs to use at least 35% of the land for green cover. My empirical exercise here will measure the stringency of FAR and GCR simultaneously.

Ideally, to estimate the shadow price measure of stringency, one would use land parcel fixed effects models to examine how adjustments of regulations affect land prices. However, such data are scarce. Instead, I use here cross-sectional data to estimate the effect of changed regulations on land prices. Despite the illustrative purpose of this exercise, one is concerned with potential omitted variables bias. For example, some unobserved local attributes (e.g., distance to employment center or natural amenity) may affect both regulated FAR and land price, resulting in a biased estimate. Following Brueckner et al. (2017), I try to mitigate this issue by controlling for a large number of "cluster" fixed effects. Specifically, I consider two or more parcels as in the same cluster if they are located in the same city district, sold in the same year, have exactly the same land-use type, and have exactly the same first 12 Chinese characters in their addresses. Thus land parcels in the same cluster are located close to each other; they should share similar local attributes.[9] If within-cluster variation in regulation is correlated with land price, then it is likely a causal effect instead of an omitted variables bias.

Due to this empirical strategy, I drop a large amount of observations that do not belong to any cluster. To guard against outliers, I exclude 1% of land parcels with the highest land prices and 1% with the lowest land prices. I also have to drop many observations because the minimum GCR requirement is missing. Ultimately, I end up with an analysis sample of 3,797 residential land parcels in 1,250 clusters (see the Appendix for descriptive statistics).

My main estimating equation is as follows:

$$r_{ij} = \alpha + \beta \cdot maxFAR_{ij} + \gamma \cdot minGCR_{ij} + \eta_j + \epsilon_{ij} \tag{5}$$

where r_{ij} is the price of land parcel i in cluster j; $maxFAR_{ij}$ is the maximum allowable FAR for land parcel i in cluster j; $minGCR_{ij}$ is the minimum allowable GCR for land parcel i in cluster j; η_j is a fixed effect for cluster j; and ϵ_{ij} is an idiosyncratic error term. If both regulations are binding, we expect that $\beta > 0$ and $\gamma < 0$. Their magnitudes indicate how stringent the two policies are; and, a one unit increase in the maximum allowable FAR is equivalent to a $\frac{\beta}{|\gamma|}$ units reduction in the minimum allowable GCR.

3.2 Results

Table 1 presents the regression results. I report both regular standard errors (in parentheses) and standard errors clustered by city (in brackets). For comparison purposes, I start by regressing land price on maximum allowable FAR and minimum allowable GCR separately in columns (1)-(2). Column (1) shows that increasing the maximum allowable FAR by one unit will increase the land price by 240 yuan per square meter of land. We can also look at this effect in relative

[9] One would prefer to define proximity using coordinates of land parcels, as in Tan et al. (2020). Unfortunately the CIA data do not provide such information.

Table 1: Estimating the stringency of land use regulations

	Dependent variable: Land price (r_{ij})			
	(1)	(2)	(3)	(4)
Maximum allowable FAR ($maxFAR_{ij}$)	240.35 (22.73)*** [119.63]**		241.00 (22.98)*** [119.90]**	113.14 (28.15)*** [62.58]*
Minimum allowable GCR ($minGCR_{ij}$)		-7.301 (5.478) [14.176]	1.040 (5.423) [11.370]	-1.329 (7.340) [11.651]
$maxFAR_{ij} * capital$				367.15 (47.74)*** [175.33]**
$minGCR_{ij} * capital$				10.964 (10.778) [22.558]
Constant (α)	Yes	Yes	Yes	Yes
Cluster fixed effect (η_j)	Yes	Yes	Yes	Yes
No. of observations	3,797	3,797	3,797	3,797
Adjusted R^2	0.927	0.924	0.927	0.929

Regular standard errors are in parentheses; standard errors clustered by city are in brackets. The analysis sample contains 1,250 clusters in 157 cities. * $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$.
Land price is measured in 2011 yuan (converted using quarterly GDP deflators) per square meter; the unit of maximum allowable FAR is 1; and the unit of minimum allowable GCR is 1 percentage point.

terms. The average land parcel in the analysis sample has a maximum allowable FAR of 2.33. If we increase it by 1% (i.e., 0.0233), the land price will increase by 5.6 yuan per square meter. Given the average land price of 2,075 yuan per square meter, this is a 0.27% increase, implying an elasticity of 0.27. Using a log-log specification and a much larger analysis sample (with fewer missing variables, while focuing on one regulation only), Brueckner et al. (2017) estimate this elasticity to be 0.36. Column (2) shows that the effect of minimum allowable GCR on land price, though negative as expected, is neither statistically nor economically significant.

Column (3) regresses land price on both maximum allowable FAR and minimum allowable GCR, as specified in equation (4). The coefficient of maximum allowable FAR is almost exactly the same as in column (1) and is still statistically significant.[10] The coefficient of minimum allowable GCR, however, is still not statistically significant. Taking the results in columns (1)-(3) together, I conclude that the FAR restrictions are binding but the GCR restrictions are not. There have been anecdotes about developers bribing local government officials in order to revise the FAR upward; this regulation has also attracted considerable attention from scholars.[11] In contrast, no attention is devoted to the GCR restrictions. This seems to be consistent with my findings here. The GCR requirement has a mean of 32.3% and a standard deviation of

[10]If the cluster fixed effects are not controlled for in column (3), the coefficient of $maxFAR_{ij}$ will increase from 241.00 to 579.48, suggesting that the fixed effect specification has indeed helped mitigate potential omitted variables bias.

[11]See, for example, Fu and Somerville (2001), Ding (2013), Brueckner et al. (2017), Cai et al. (2017), and Tan et al. (2020).

6.80 percentage points. Perhaps most developers would want to leave that much land for green cover in order to attract buyers and maximize profit, thus it is reasonable to see that the GCR requirement is not binding.[12]

Lastly, I construct a dummy variable *capital* to indicate a province capital or a direct-control municipality. In my analysis sample, 35.7% of the land transactions are in these cities. Since these cities are larger and more densely populated, one would expect them to have more stringent land use regulations. In column (4), I interact $maxFAR_{ij}$ and $minGCR_{ij}$ with the *capital* dummy, allowing the stringency to be different in these cities. Indeed, in cities that are not province capitals or direct-control municipalities, a one unit increase in maximum allowable FAR will increase the land price by 113 yuan per square meter; in province capitals and direct-control municipalities, this effect will be 367 yuan higher. This implies that the FAR restrictions are much more stringent in province capitals and direct-control municipalities. On the other hand, neither the minimum allowable GCR nor its interaction with the *capital* dummy is statistically significant, suggesting that the GCR requirement is not binding in either group of cities.[13]

3.3 Discussion

Before closing, I would like to comment on the limitations of this empirical approach to estimating the shadow price measure.

First, this approach faces serious challenges of endogeneity. Parcel-specific land use regulations, such as the ones studied here, almost surely vary with unobserved local attributes that affect land price. Also, there might be unobserved regulations correlated with the observed ones. Thus a regression of land prices on regulation levels likely suffers from an omitted variables bias. Following Brueckner et al. (2017), I have estimated a "cluster" fixed effects model to mitigate the potential bias. However, one wonders whether this is enough. An IV estimate may be preferred, yet finding a valid IV is extremely difficult.[14]

As implied by equation (3), one way to address this endogeneity issue could be to estimate an equivalent regulatory tax measure. This strategy is feasible when $R'(k(h)) \cdot k'(h)$ is known or can be reliably estimated. For example, if a binding floor area ratio ($h = \bar{h}$) is the only land use restriction, then $R'(k(h)) \cdot k'(h) = 1$ and the shadow price measure and the regulatory tax measure are exactly the same. In cases like this, one could estimate either measure depending on data availability and reliability. When both measures are estimable, they can be used to cross check each other. As demonstrated by Glaeser et al. (2005), location-specific housing price data

[12] As shown in the descriptive statistics table in the Appendix, the within-cluster variation of the GCR requirement is rather small. So it is also possible that I do not have enough statistical power to precisely estimate its effect.

[13] As a robustness check, I rerun the regressions in Table 1 using log land price as the dependent variable. The results are similar: Whereas the maximum allowable FAR always has a statistically significant positive effect on log land price, the minimum allowable GCR never has a statistically significant negative effect.

[14] Land use regulations are often introduced, revised, and removed; they vary significantly across boundaries. Such discrete changes may create some contexts where other empirical strategies (such as event studies, regression discontinuity, or difference-in-differences design) can be used to address the endogeneity concerns (see, e.g., Nakajima and Takano 2021).

is easily available, and construction cost estimates can be obtained from commercial providers. With these two sources of information, they compute the regulatory tax as price markups over construction costs for New York and 21 other cities. They show that the regulatory tax can be as high as 50% of the housing price on Manhattan and that this number ranges from one third to one half for major California cities. Note that the information they used for the regulatory tax measure is entirely different from what I have used to estimate the shadow price measure. Their calculations are free of endogeneity concerns.

Second, my estimation of land use stringencies in Table 1 assumes no interaction between the two types of regulations. Specifically, I have assumed that the stringency of one regulation does not depend on the other regulation. In reality, of course, this assumption might be violated. For example, a higher green coverage requirement may make an FAR requirement more stringent. In that case, the two constraints in the developer's profit maximization problem may be written as $R_1(k, \bar{R}_2) \leq \bar{R}_1$ and $R_2(k, \bar{R}_1) \leq \bar{R}_2$. However, the Lagrangian multiplier associated with a constraint cannot be directly interpreted as the stringency of the regulation. In contrast, when there are multiple land use regulations, the regulatory tax measure by Glaeser et al. (2005) treats the amount of built housing as the result of all relevant regulations, allowing for arbitrary interactions between different regulations. The regulatory tax has this flexibility because it is not meant to measure the stringency of any particular regulation.

It is worth noting that my assumption of no interaction has more implications. In particular, if different regulations affect the developer's profit independently, then it is straightforward to extend equation (3) to the multiple regulation case:

$$p - C'(h) = \sum_i \lambda_i^* R_i'(k(h)) \cdot k'(h). \tag{3'}$$

That is, the regulatory tax measure is a linear combination of the shadow prices of different specific regulations. This relationship can be exploited empirically in two different ways. First, if λ_i^* and $R_i'(k(h)) \cdot k'(h)$ can be reliably estimated for all i, then one can calculate the regulatory tax by equation (3'). Second, when $p - C'(h)$ and λ_i^* can be reliably estimated, one can regress $p - C'(h)$ on all λ_i^* to explore each regulation's contribution to the regulatory tax. The coefficient of each λ_i^* can be interpreted as $R_i'(k(h)) \cdot k'(h)$, which indicates how this particular regulation affects the quantity of housing on one unit of land.

Finally, this approach assumes perfect competition. In the developer's profit maximization problem, I assumed perfect competition on both housing and capital markets. This is not crucial. I can relax it by replacing housing price p with $p(h(k))$ and capital price i with $i(k)$. Nothing is affected theoretically or empirically. The envelope theorem still applies and the Lagrangian multiplier has the same interpretation: It represents the loss of the developer's profit as the regulation is tightened by one marginal unit. To proceed empirically, I also assumed perfect competition on the land market, so that land price captures all of the developer's pre-rent profit. This assumption, although commonly made in the literature (see Fu and Somerville 2001, Brueckner et al. 2017, and Tan et al. 2020), is not entirely innocuous. In the context of China, urban land parcels are auctioned off to developers. The maximum pre-rent profit can be thought of as the value of the land parcel to a developer. If we model the land bidding process as an English auction (which is indeed commonly used in China), then the selling price is the second highest value among all the bidders. Thus the estimated shadow price measure reflects the effect of a tightened regulation on the second highest value. Whether this is a good proxy for the real shadow price (the winning developer's willingness to pay to avoid a marginal tightening of the regulation) depends crucially on the heterogeneity and the number of competing developers.

4 Conclusion

I propose a shadow price measure of land use stringency that equals the developer's willingness to pay for a marginal relaxation of the regulation. This measure is simple to formulate, has a clear theoretical interpretation, and can be easily extended to deal with multiple regulations. It provides a unified framework for understanding three existing theory-based measures of land use stringency. Given the advantages of this shadow price measure, it may be useful for future empirical research.

Appendix

Table A.1 presents descriptive statistics for key variables used in the regression analysis. There are 3,797 observations and 1,250 clusters. For the three continuous variables, the Stata command *xtsum* is used to decompose the total variation into between and within variation.

Table A.1: Descriptive statistics

Variable		Mean	Std Dev	Min	Max
Land price		2,074.99	2,548.72	101.87	2,3872.58
	between		2,633.10		
	within		575.31		
Maximum allowable FAR		2.333	1.231	0.1	16
	between		1.085		
	within		0.491		
Minimum allowable GCR (%)		32.288	6.799	0	75
	between		6.116		
	within		2.081		
Province capital		0.357	0.479	0	1

References

Albouy, David, and Gabriel Ehrlich. 2018. "Housing Productivity and the Social Cost of Land-Use Restrictions." *Journal of Urban Economics* 107, 101-120.

Brueckner, Jan K., Shihe Fu, Yizhen Gu, and Junfu Zhang. 2017. "Measuring the Stringency of Land-Use Regulation: The Case of China's Building-Height Limits." *Review of Economics and Statistics* 99, 663-677.

Brueckner, Jan K., and Ruchi Singh. 2020. "Stringency of Land-Use Regulation: Building Heights in US Cities." *Journal of Urban Economics* 116, 103239.

Cai, Hongbin, Zhi Wang, and Qinghua Zhang. 2017. "To Build above the Limit? Implementation of Land Use Regulations in Urban China." *Journal of Urban Economics* 98, 223-233.

Cheshire, Paul C., and Christian A. L. Hilber. 2008. "Office Space Supply Restrictions in Britain: The Political Economy of Market Revenge." *Economic Journal* 118(529), pp. F185-F221.

Cheung, Ron, Keith Ihlanfeldt, and Thomas Mayock. 2009. "The Regulatory Tax and House Price Appreciation in Florida." *Journal of Housing Economics* 18(1), 34-48.

Ding, Chengri. 2013. "Building Height Restrictions, Land Development and Economic Costs." *Land Use Policy* 30, 485-495.

Fu, Shihe, Xiaocong Xu, and Junfu Zhang. 2021. "Land Conversion across Cities in China." *Regional Science and Urban Economics* 87, 103643.

Fu, Yuming, and C Tsuriel Somerville. 2001. "Site Density Restrictions: Measurement and Empirical Analysis." *Journal of Urban Economics* 49(2), 404-423.

Furth, Salim. 2021. "Foundations and Microfoundations: Building Houses on Regulated Land." Mercatus Center Working Paper, George Mason University.

Glaeser, Edward, and Joseph Gyourko. 2018. "The Economic Implications of Housing Supply." *Journal of Economic Perspectives* 32(1), 3-30.

Glaeser, Edward L., Joseph Gyourko, and Raven Saks. 2005. "Why is Manhattan So Expensive? Regulation and the Rise in Housing Prices." *Journal of Law and Economics* 48, 331-369.

Glaeser, Edward L, and Bryce A Ward. 2009. "The Causes and Consequences of Land Use Regulation: Evidence from Greater Boston." *Journal of Urban Economics* 65 (3), 265-278.

Gyourko, Joseph, Jonathan S. Hartley, and Jacob Krimmel. 2021. "The Local Residential Land Use Regulatory Environment across U.S. Housing Markets: Evidence from a New Wharton Index." *Journal of Urban Economics* 124, 103337.

Gyourko, Joseph, and Jacob Krimmel. 2021. "The Impact of Local Residential Land Use Restrictions on Land Values across and within Single Family Housing Markets." *Journal of Urban Economics* 126, 103374.

Gyourko, Joseph, Albert Saiz, and Anita Summers. 2008. "A New Measure of the Local Regulatory Environment for Housing Markets: The Wharton Residential Land Use Regulatory Index." *Urban Studies* 45(3), 693-729.

Huang, Haifang, and Yao Tang. 2012. "Residential Land Use Regulation and the US Housing Price Cycle between 2000 and 2009." *Journal of Urban Economics* 71, 93-99.

Ihlanfeldt, Keith R. 2007. "The Effect of Land Use Regulation on Housing and Land Prices." *Journal of Urban Economics* 61(3), 420-435.

Kok, Nils, Paavo Monkkonen, and John M. Quigley. 2014. "Land Use Regulations and the Value of Land and Housing: An Intra-Metropolitan Analysis." *Journal of Urban Economics* 81, 136-148.

Levine, Ned. 1999. "The Effects of Local Growth Controls on Regional Housing Production and Population Redistribution in California." *Urban Studies* 36(12): 2047-2068.

Malpezzi, Stephen. 1996. "Housing Prices, Externalities, and Regulation in U.S. Metropolitan Areas." *Journal of Housing Research* 7(2), 209-241.

Mayer, Christopher J., and C. Tsuriel Somerville. 2000. "Land Use Regulation and New Construction." *Regional Science and Urban Economics* 30(6), 639-662.

Moon, Byunggeor. 2019. "The Effect of FAR (Floor Area Ratio) Regulations on Land Values: The Case of New York." *Papers in Regional Science* 98(6), 2343-2354.

Nakajima, Kentaro, and Keisuke Takano. 2021. "Estimating the Impact of Land Use Regulation on Land Price: At the Kink Point of Building Height Limits in Fukuoka." RIETI Discussion Paper Series 21-E-88, The Research Institute of Economy, Trade and Industry.

Pollakowski, Henry O., and Susan M. Wachter. 1990. "The Effects of Land-Use Constraints on Housing Prices." *Land Economics* 66(3), 315-324.

Quigley, John M, and Larry A Rosenthal. 2005. "The Effects of Land Use Regulation on the Price of Housing: What Do We Know? What Can We Learn?" *Cityscape* 8(1), 69-137.

Sunding, David L., and Aaron M. Swoboda. 2010. "Hedonic Analysis with Locally Weighted Regression: An Application to the Shadow Cost of Housing Regulation in Southern California." *Regional Science and Urban Economics* 40(6), 550-573.

Tan, Ya, Zhi Wang, and Qinghua Zhang. 2020. "Land-Use Regulation and the Intensive Margin of Housing Supply." *Journal of Urban Economics* 115, 103199.

Turner, Matthew, Andrew Haughwout, and Wilbert Van der Klaauw. 2014. "Land Use Regulation and Welfare." *Econometrica* 82(4), 1341-1403.

van Soest, Daan P., John A. List, and Tim Jeppesen. 2006. "Shadow Prices, Environmental Stringency, and International Competitiveness." *European Economic Review* 50(5), 1151-1167.