DISSERTATION PROPOSAL

PhD in Business

Essays on Hedge Fund Replication

Methodological Assessment and
Development of the Factor Approach,
Model Selection, Nonlinear Modeling and Policy Perspectives

Guillaume Weisang
Bentley University

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Thank you!

Chair

Dominique M. Haughton
Professor of Mathematical Sciences
Bentley University, MA

Committee

Victoria R. Steblovskaya	Thierry Roncalli	José M. Marín Vigueras
Associate Professor	*Head of R&D,*	*Professor of Finance*
of Mathematical Sciences	*Lyxor Asset Management*	*IMDEA Social Sciences*
Bentley University, MA	and	*Madrid, Spain*
	Professor of Finance,	
	Université of Évry, France	

Outline

Part I

Background

Investing in the Hedge Fund Industry

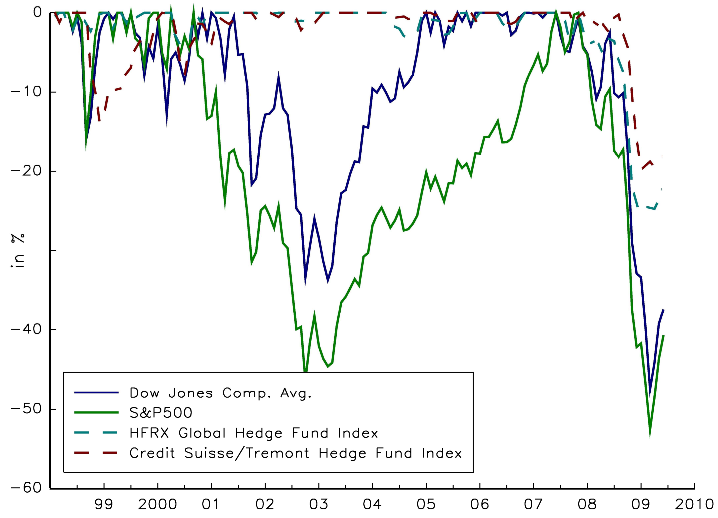

Figure: Drawdown graph (represented with $(-)$ sign): January 1998 to May 2009

Characteristics of HF management

- active management
- non traditional assets (e.g., derivatives)

HF returns

- heavy tails (tail risk)
- non linear w.r.t. stock markets

Definition (Drawdown)

A measure of the decline from a historical peak in the cumulative profit $X(t)$ of a financial trading strategy.

Formally, one can write

$$D(T) = \text{Max}\left\{0,\ \underset{t\in(0,T)}{\text{Max}}\ X(t) - X(T)\right\}.$$

A Few Facts About Hedge Funds

(Why Hedge Fund Replication Has Become Very Important[1])

Hedge Funds

1. Manager is best judge of appropriate risk/reward trade-off
2. Highly proprietary trading strategies
3. Ultimate objective: return
4. Risk management is not central to the success of the Hedge Fund
5. Regulation and compliance $\cong$ drag on performance
6. Little intellectual property in the fund:
   the general partner *is* the fund

Institutional Investors

1. As fiduciaries, for each HF manager, institutions need to understand
   1.1 investment process
   1.2 risk exposures
2. Risk management and risk transparency are essential
3. Highly regulated environment
4. Institutions desire structure, stability, and consistency

Definition (Hedge Fund)

A Hedge Fund (HF)

- is an investment fund
- is open to a limited range of investors
- is permitted by regulators to undertake a wider range of investment and trading activities than other investment funds
- pays a performance fee to its investment manager

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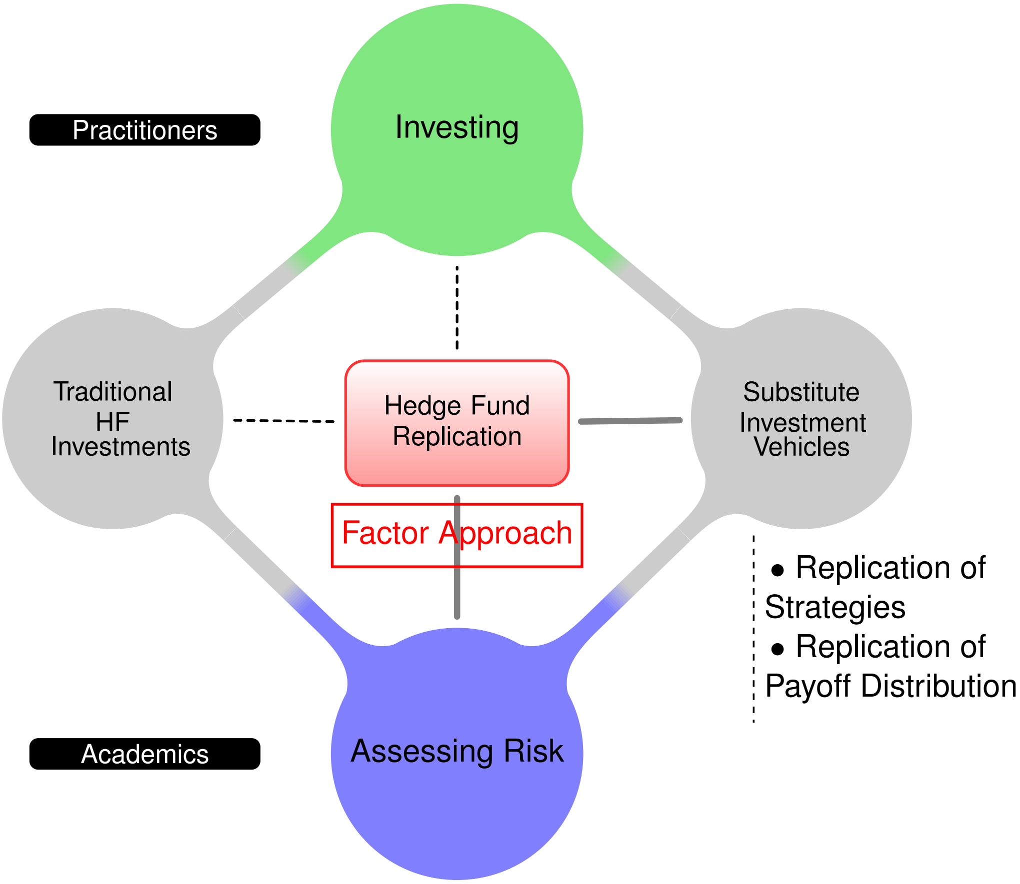
[1] The following has been excerpted from [Lo, 2008].

Hedge Fund Replication

As a Tool



Practitioners

Investing

Traditional HF Investments

Hedge Fund Replication

Substitute Investment Vehicles

Factor Approach

- Replication of Strategies
- Replication of Payoff Distribution

Academics

Assessing Risk

Overview of the Factor Approach

A hedge fund portfolio:

$$r_t^{\text{HF}} = \sum_{i \in I} \omega_{it} r_{it}$$

asset i's return

HF return

Assumption

The structure of all asset returns can be summarized by a set of risk factors $\{F_j\}_{j=1,\ldots,m}$:

$$\forall t \qquad r_{it} = \alpha_i + \sum_{j=1}^{m} \beta_{ij} F_{jt} + \xi_{it}$$

asset i's return

risk factor

with

$$\mathbb{E}\left(\xi_{it} \left| F_{1t} \ldots F_{mt}\right.\right) = 0$$

A typical factor model

One assumes

$$r_t^{\text{HF}} = \alpha_t^{\text{HF}} + \sum_{j=1}^{m} w_{jt}^{\text{HF}} F_{jt} + \varepsilon_t$$

such that

$$\alpha_t^{\text{HF}} = \sum_{i \in I} \alpha_i \omega_{it}$$

risk exposures

$$w_{jt}^{\text{HF}} = \sum_{i \in I} \omega_{it} \beta_{ij}$$

$$\varepsilon_t = \sum_{i \in I} \omega_{it} \xi_{it}$$

Skip Detailed Literature Review

Literature Review of the Factor Approach

Overview

- Linear models
  [Fung and Hsieh, 1997, Amenc et al., 2007, Hasanhodzic and Lo, 2007]
  - Linear and non linear factors depending on the type of strategies followed by hedge funds.
    e.g., Convertible and Fixed Income Arbitrage, Event Driven, Long/Short Equity, etc.
  - Factor Selection: More factors to improve in-sample (and out-sample) fit?
- (Static) option-based models [Diez de los Rios and Garcia, 2008]

$$r_t^{\text{HF}} = \sum_{j=1}^{m} w_j^{\text{HF}} F_{jt} + w_{m+1}^{\text{HF}} \max(F_{1t} - s_t, 0) + \varepsilon_t$$

  - only for risk assessment
  - mostly academic exercises

Literature Review of the Factor Approach

Estimation procedures

- Traditionally, estimation and calibration procedures (in chronological order)
  1. Full factor model OLS regressions
  2. Stepwise procedures (versus economic selection of factors)
  3. Rolling-windows OLS (to try to capture dynamic allocation)
- More recently, state-space modeling has been introduced to model and estimate HF returns
  - Markov Regime-Switching Model [Amenc et al., 2008]
  - Kalman Filter [Roncalli and Teiletche, 2008]

Literature Review of the Factor Approach

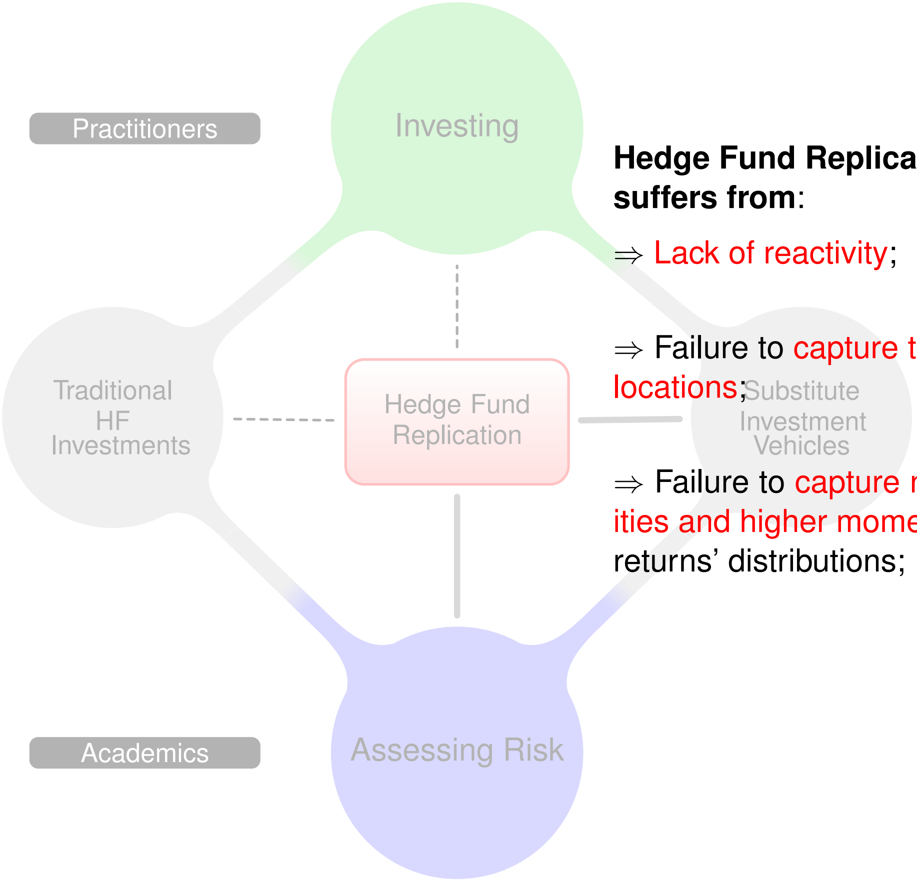
Summary

▸ Go to 'Overview of the Factor Approach'

- Static Linear factor models [Amenc et al., 2007]
  - Lack reactivity
  - Fail the test of robustness, giving poor out-of-sample results
- Factor selection [Fung and Hsieh, 1997] [Lo, 2008]
  - In static models, **economic selection** of factors ⟶ significant improvement over other methodologies for out-of-sample robustness test.
  - In dynamic models, [Darolles and Mero, 2007] uses a PCA-based factor evaluation methodology [Bai and Ng, 2006] on rolling OLS regressions.
    - Improvement over "naive" inclusion of all relevant economic factors
    - Poor Interpretability of the evaluated factors
- Dynamic linear models [Roncalli and Teiletche, 2008] [Lo, 2008] [Jaeger, 2009]:
  Capturing the *unobservable* dynamic allocation using **traditional (OLS) methods** is
  - Very difficult
  - Estimates can vary greatly at balancing dates
- Nonlinear models ⇌ methodological challenge [Amenc et al., 2008] [Diez de los Rios and Garcia, 2008]

Hedge Fund Replication

As a Tool



Hedge Fund Replication suffers from:

⇒ Lack of reactivity;

⇒ Failure to capture tactical allocations;

⇒ Failure to capture nonlinearities and higher moments in HF returns' distributions;

Research Program

4 Objectives

I investigate the case of Dynamic factor models

1. Provide a new perspective on HFR by inscribing it in a more general framework
    - Fresh perspective (in continuity with analytic approach to finance)
    - New tools (developed in other fields – e.g., engineering – to address similar problems)
2. Model and factors selection
    - Develop and adapt the associated methodology, specifically in relation with building a replicating portfolio with implementable factors
3. Nonlinearities in HF returns
    - Assess the problem in the case of dynamic replication
    - If necessary, develop the tools for the replication of nonlinearities
        - 3.1 Inclusion of nonlinear functions (very difficult)
        - 3.2 Develop a robust methodology
4. Policy and regulation perspectives
    - examine the perspectives that HFR AND related quantitative approaches can offer for the regulatory framework of the HF industry.
        - 4.1 *Operational* Due Diligence
        - 4.2 Risk assessment at the industry level?

Tracking Problems

Definition (Tracking Problem)

The following two equations define a tracking problem (TP) [Arulampalam et al., 2002]:

$$\left\{\begin{array}{lcll} \mathbf{x}_k & = & f(t_k, \mathbf{x}_{k-1}, v_k) & \text{(Transition Equation)} \\ \underset{\substack{\uparrow \\ \text{“shadow”}}}{\mathbf{z}_k} & = & h(t_k, \underset{\substack{\uparrow \\ \text{object}}}{\mathbf{x}_k}, \eta_k) & \text{(Measurement Equation)} \end{array}\right.$$

where

- $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is the state vector, and $\mathbf{z}_k \in \mathbb{R}^{n_z}$ the measurement vector at step k.
- v_k et η_k are mutually independent i.i.d noise processes.
- The functions f and h can be non-linear functions.

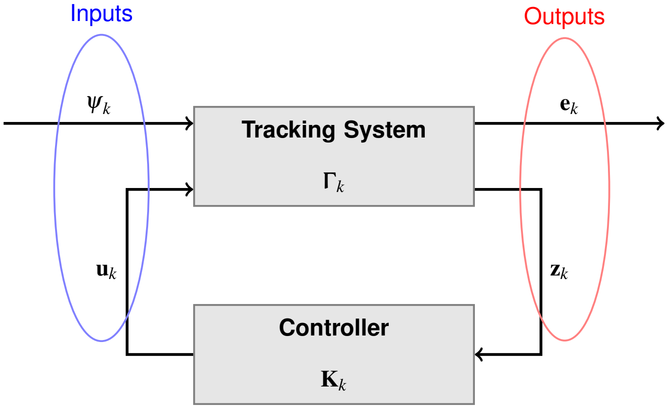
Tracking Problems and Tactical Allocation

Tracking Systems

HFR: $\mathbf{x}_k = (w_{1k}^{\text{HF}}, \ldots, w_{mk}^{\text{HF}})'$

Discrete case, at time step k

- **Outputs**
  - Tracking Error
    $\mathbf{e}_k = \mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}$
    $\mathbf{e}_k = r_k^{\text{HF}} - r_k^{\text{Clone}}$
  - Censored measurement $\mathbf{z}_k$
    $\mathbf{z}_k = r_k^{\text{HF}}$
- **Inputs**
  - Exogenous signals
    $\psi_k = (\mathbf{x_0}, \eta_{1:k}, \nu_{1:k})$
    HF changes in allocation, strategies or reporting
  - Controlled input $\mathbf{u}_k$
    Assumption: $\mathbf{u}_k = K_k\,\mathbf{z}_k$
    Adjustments to the replication portfolio's risk exposures



Bayesian Filters

Optimal Control Theory

Under some general assumptions, one can prove

▸ Go to proof

$$\text{tracking error} \rightarrow \mathbf{e}_k = \mathbf{T}_{e\psi_k}\,\psi_k \leftarrow \text{exogenous signals}$$

with

$$\underset{\uparrow \atop \text{transfer function}}{\mathbf{T}_{e\psi_k}} = \Gamma_{e\psi_k} + \Gamma_{eu_k}\underset{\uparrow \atop \text{Controller } K_k}{K_k}\left(I - \Gamma_{zu_k}\right)^{-1}\Gamma_{z\psi_k}$$

The role of the controller K_k is to

- stabilize the system
- make $\mathbf{T}_{e\psi}$ small in an appropriate sense.

Bayesian Filters are algorithms which provide the **optimal estimators** of the state $\mathbf{x}_k$

Definition (Stability)
A system is said to be marginally stable if the state $\mathbf{x}$ is bounded for all time t and for all bounded initial states $\mathbf{x}_0$.

Bayesian Filters

Solving Tracking Problems

- Prediction equation

$$p\left(\mathbf{x}_k \mid \mathbf{z}_{1:k-1}\right) = \int p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}\right) p\left(\mathbf{x}_{k-1} \mid \mathbf{z}_{1:k-1}\right) \mathrm{d}\mathbf{x}_{k-1}$$

- Update equation

$$p\left(\mathbf{x}_k \mid \mathbf{z}_{1:k}\right) \propto p\left(\mathbf{z}_k \mid \mathbf{x}_k\right) p\left(\mathbf{x}_k \mid \mathbf{z}_{1:k-1}\right)$$

- Best estimates

$$\hat{\mathbf{x}}_{k|k-1} = \mathbb{E}\left[\mathbf{x}_k \mid \mathbf{z}_{1:k-1}\right] \quad \hat{\mathbf{x}}_{k|k} = \mathbb{E}\left[\mathbf{x}_k \mid \mathbf{z}_{1:k}\right]$$

- Implementation ▸ Go to GTAA example ▸ Go to figure
  - Kalman Filter (KF): linear Gaussian case
  - H_∞ Filters or Particle Filters (PF): nonlinear or non Gaussian case

Example (Random Walk)

$$\begin{cases} \mathbf{x}_k & = & \mathbf{x}_{k-1} & \pm 1 \\ \mathbf{z}_k & = & \mathbf{x}_k & \pm 1 \end{cases}$$

$$\mathbf{x}_0 = 1/2$$

Prediction

$$\hat{\mathbf{x}}_{1|0} = \begin{cases} -1/2 & p = & 1/2 \\ 3/2 & q = & 1/2 \end{cases}$$

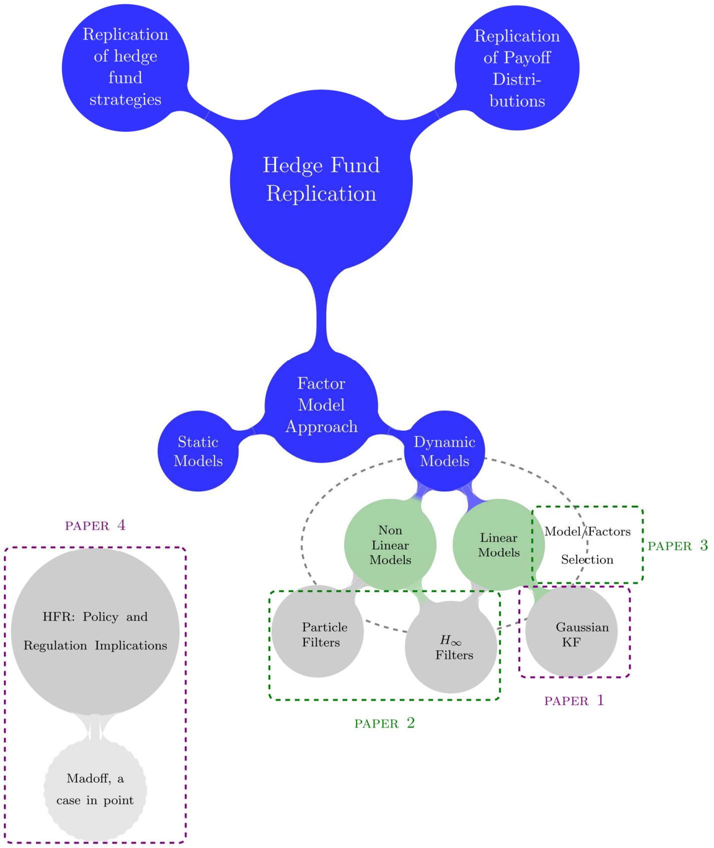
Update $\mathbf{z}_1 = -3/2$

$$\mathbf{x}_1 \mid \mathbf{z}_1 = \begin{cases} -1/2 & p = & 1 \\ 3/2 & q = & 0 \end{cases}$$

Estimate

$$\hat{\mathbf{x}}_{1|1} = (1)\left(-\frac{1}{2}\right) + (0)\left(-\frac{3}{2}\right) = -\frac{1}{2}$$

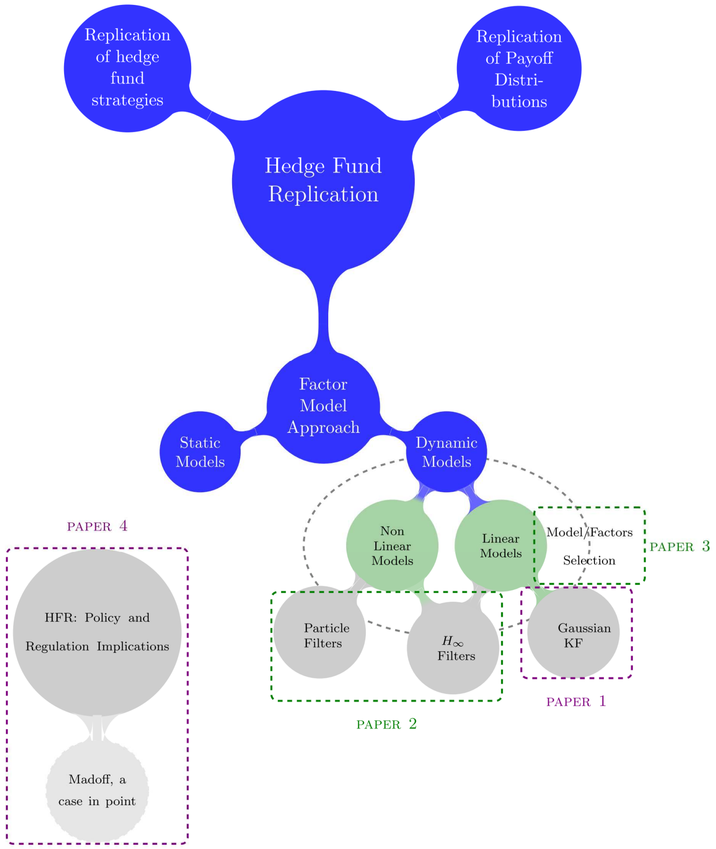
Dissertation Conceptual Map



Part II

Dissertation Papers

Dissertation Conceptual Map



Paper 1: HFR – Gaussian Linear Case

Objectives

Gaussian Linear Case

$$\begin{cases} \mathbf{w}_k^{\text{HF}} & = & \mathbf{w}_{k-1}^{\text{HF}} + v_k \\ r_k^{\text{HF}} & = & \mathbf{r}_k' \mathbf{w}_k^{\text{HF}} + \eta_k \end{cases}$$

with v_k and η_k *i.i.d.* Gaussian noise processes

The objectives of *Paper 1* are to review and promote the use of KF

- Understand how the KF algorithm adjusts to changes in HF dynamic
- Show that KF provides sensible “explanations”
- Look into the **alpha** replication problem

Definition (Alpha)
The ***alpha*** is a measure of the risk-adjusted performance of an asset.
In the case of HF, the ***alpha*** is considered to represent the “talent” of the manager.

Paper 1: HFR – Gaussian Linear Case

Summary of Results

Key points

- Describe in terms of investment decisions the KF’s adjustments to the replicating portfolio
- Provide a detailed example with economic interpretation
- Show that Core/Satellite approach to HFR can provide access to the “alpha”

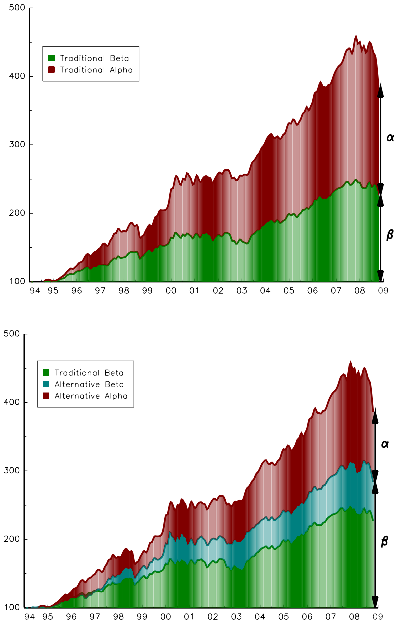
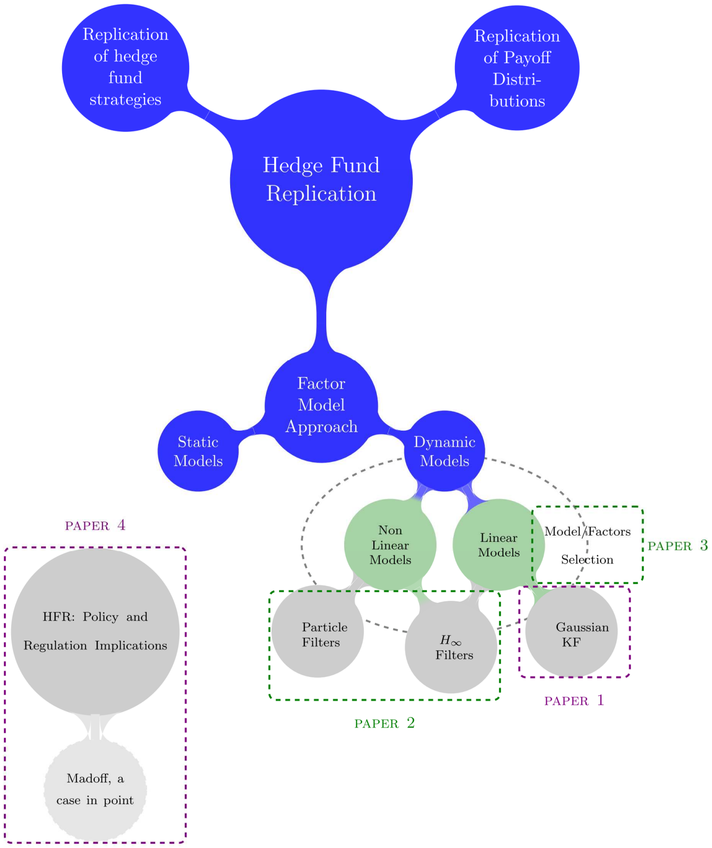


Figure: Top: traditional replication; Bottom: KF replication

Dissertation Conceptual Map



Replication of hedge fund strategies

Replication of Payoff Distributions

Hedge Fund Replication

Factor Model Approach

Static Models

Dynamic Models

PAPER 4

Non Linear Models

Linear Models

Model/Factors Selection

PAPER 3

HFR: Policy and Regulation Implications

Particle Filters

H_∞ Filters

Gaussian KF

PAPER 1

PAPER 2

Madoff, a case in point

Hedge Fund Replication: The Nonlinear Non-Gaussian Case

Why It is Interesting

- HF Returns are not Gaussian
    - negative skewness and positive excess kurtosis.
- Nonlinearities in HF Returns
    - Nonlinearities documented from the very start of hedge-fund replication – see, e.g., [Fung and Hsieh, 1997].
    - Nonlinearities
        - are important for some strategies but not for the entire industry [Diez de los Rios and Garcia, 2008].
        - may be due to positions in derivative instruments or un-captured dynamic strategies – see, e.g., [Merton, 1981].
    - No successful hedge fund replication using non-linear models has ever been done

Essays on HFR
G. Weisang
Paper 1: Gaussian Linear Case
Paper 2: Non-Gaussian Nonlinear Case
Objectives
Findings: Key Points and Future Developments
Paper 3: Model and Factors Selection
Paper 4: Policy and Regulation Implications
Future Perspectives

Paper 2: HFR – Non-Gaussian Nonlinear Case

Objectives

Non-Gaussian or Nonlinear Case

Non Gaussian

$$\left\{\begin{array}{lcl} \mathbf{w}_k^{\text{HF}} & = & \mathbf{w}_{k-1}^{\text{HF}} + \nu_k \\ r_k^{\text{HF}} & = & \mathbf{r}_k' \mathbf{w}_k^{\text{HF}} + \eta_k \\ \eta_k & \sim & \mathscr{H} \end{array}\right.$$

with $\mathscr{H}$ non Gaussian

Nonlinear

$$\left\{\begin{array}{lcl} \mathbf{w}_k^{\text{HF}} & = & \mathbf{w}_{k-1}^{\text{HF}} + \nu_k \\ r_k^{\text{HF}} & = & \mathbf{r}_k' \mathbf{w}_k^{\text{HF}} \\ & + & w_{k-1,(m+1)}^{\text{HF}} r_{k,(m+1)}(s_k) + \eta_k \end{array}\right.$$

with $r_{k,(m+1)}(s_k)$ nonlinear

Option on S&P 500

$\Longrightarrow$ May be solved (approximations) using Particle Filters or H_∞ Filters.

The objectives of *Paper 2* are to

- explore the nature of HF nonlinearities
    1. non Gaussian errors
    2. non linear factor
- explore possible remedy: PF
- develop HFR methodology robust to violation of Gaussian and linear hypotheses: H_∞ Filters

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Paper 2: HFR – Non-Gaussian Nonlinear Case

Key points and Future Developments

Gaussian assumption KF's tracking errors have skew and excess kurtosis. A remedy: Skew t distribution

1. very difficult direct estimation of parameters in PF
2. no luck with two-step procedure (KF + GMM) $\Longrightarrow \searrow$ Skew, $\nearrow$ TE

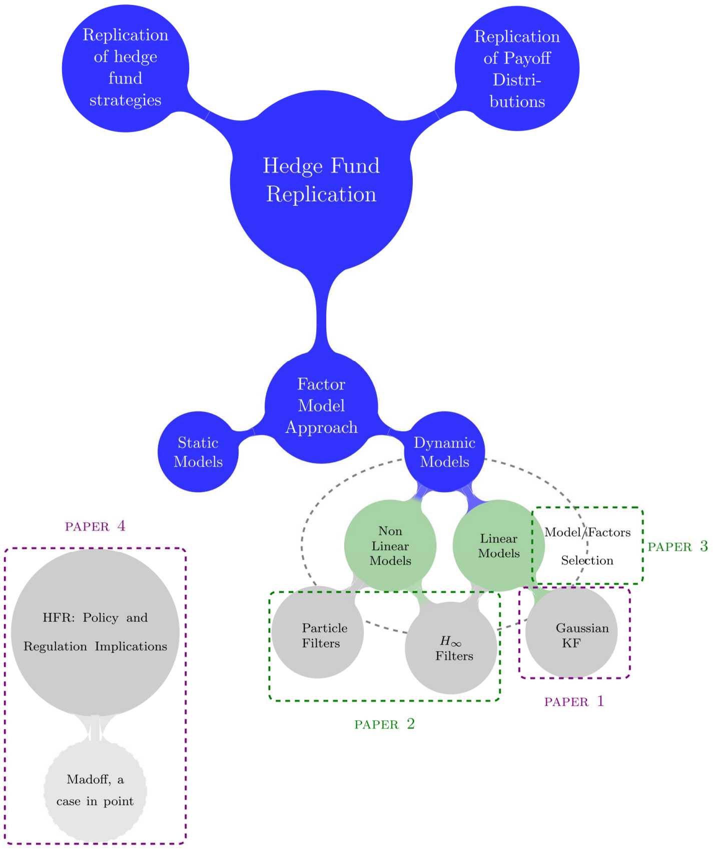
Nonlinear Factor Endogenous and exogenous

1. Exogenous factors are extremely data dependent
2. Endogenous factors: some success using a grid-based approach and KF; PF code has to be parallelized
3. For now, purely academic exercise

Robust Methodology

1. TO BE DEVELOPED
2. H_∞ Filters minimize worst cases $\longrightarrow$ robust to violations of Gaussian and linearity assumptions

Dissertation Conceptual Map



Paper 3: HFR – Model and Factors Selection

The Problem of Factors Selection

Two questions about adding or deleting a factor:

1. Improvement of the performance of the replication ?
2. Pertinence ? (risk management $\neq$ HF tracker)

		$\hat{\mu}_{1Y}$	π_{AB}	σ_{TE}	ρ	τ	ρ_S
	6F	7.55	75.93	3.52	87.35	67.10	84.96
+	**CREDIT**	7.35	73.91	3.51	87.46	67.30	85.11
+	**GSCI**	7.46	75.07	3.55	87.42	68.74	86.52
+	**VIX**	6.55	65.94	4.05	83.71	67.29	85.14
+	**BUND**	7.75	77.94	3.54	87.09	66.95	84.84
+	**JPY/USD**	7.37	74.18	3.56	87.02	66.42	84.23
+	**USD/GBP**	7.48	75.25	3.58	86.81	66.66	84.63
+	**MXEF/SPX**	7.56	76.06	3.03	90.68	72.92	89.94
–	**SPX**	6.42	64.56	6.31	47.51	32.19	45.82
–	**RTY/SPX**	7.08	71.20	4.66	75.92	54.02	73.55
–	**SX5E/SPX**	6.51	65.47	3.73	85.88	68.19	85.94
–	**TPX/SPX**	7.34	73.82	3.72	85.78	64.43	82.30
–	**UST**	7.86	79.13	3.50	87.47	66.92	84.79
–	**EUR/USD**	6.57	66.08	3.60	86.59	66.66	84.70
	7F	7.82	78.64	3.05	90.55	72.92	89.95

Paper 3: HFR – Model and Factors Selection

Literature Review

Problem	Theoretical Question	Practical Question	Solution
Model Selection	Is the model optimal?	How many parameters?	Information Criterion, e.g., [Akaike, 1974, Cavanaugh, 1997]
		Factor models: How many factors? Exact or approximate factor structure?	Approximate factor structures and Dynamic (Asymptotic) Principal Components, e.g., [Connor and Korajczyk, 1986, Stock and Watson, 2002, Hallin and Liška, 2007]
Factors Selection	Which factors?	Estimation Identification	Inferential theory for static and dynamic factor models, e.g., [Bai and Ng, 2006]
Variables Selection	Which variables?	Representation of factors for (portfolio) implementation	Application of [Bai and Ng, 2006] to HFR [Darolles and Mero, 2007]

Same Literature!

Paper 3: HFR – Model and Factors Selection

Objectives

Factors Estimation Using the covariance matrix a vast set of assets (including individual HFs if possible)

1. Estimate all possible factors with PCA on rolling time-windows $[k-T,\dots,k]$ (cf. [Darolles and Mero, 2007])
2. Estimate the number of factors $\hat{m}$ using [Bai and Ng, 2006]'s tests

Factor Exposures Estimation Once the factors selected and estimated,

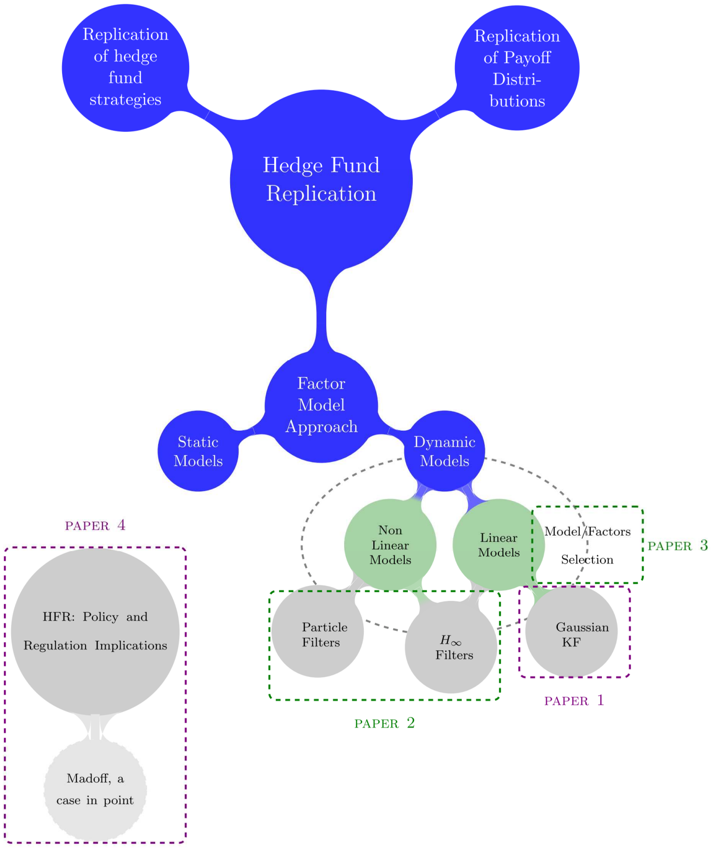
- KF or H_∞ Filter to estimate the replication exposures $\hat{\mathbf{w}}^{\text{HF}}_{k+1|k}$ (instead of OLS regressions as in [Darolles and Mero, 2007]).

Factors' interpretation and implementation: 2 possibilities

1. Statistical identification using [Bai and Ng, 2006]'s tests in the spirit of [Darolles and Mero, 2007]
2. **Online** tracking method for MIMO problems [Kim et al., 2004, Kim et al., 2007]: *track the estimated factors (output) with observable and investable assets or indices (input).*

Definition (Online)
Online refers to a recursive method of tracking using observations in a sequential manner, as they become available.

Dissertation Conceptual Map



Paper 4: Policy and Regulation Implications

Motivation

Recall Slide 2: HF managers vs. Investors

1. Market risks are borne by investors, not by fund manager.
2. The fund manager is the only decision maker.

Questions

- How can the information asymmetry between the fund manager and investors be reduced?
  - Agency problems
- How may investors have control over the fund manager? How can regulators ensure investors protection?

Madoff, a case in point

- Massive fraud: $\sim \$60$ Bn
- Some individual but also some institutional investors
- Lots of red flags
  - some operational (e.g., broker, custodian and fund manager)
  - some quantitative: related to the statistics of the fund

Paper 4: Policy and Regulation Implications

Outline of the paper

1. Madoff's story
2. Understand how Madoff lost the capital
    - Understand Madoff investment strategy
    - Explain Madoff's collapse: developed a model of a Ponzi scheme in asset management industry
3. New lessons for operational risk capital requirements
4. Implications of the Madoff case for regulators and the investment industry
    - Rethinking due diligence processes
    - Future of the HF industry

Paper 4: Policy and Regulation Implications I

Main findings

Understand how Madoff lost the capital

- Understand Madoff investment strategy
    1. Bull-Spread strategy: extremely attractive in theory
    2. In practice, to obtain these ideal results, we need a very good stock picking process
        - systematic outperformance with respect to the index
        - perfect correlation with the index
- Explain Madoff's collapse: Ponzi scheme model

- Capital shrinkage: Management fees = main contributors.
- Default may be avoided only if management fees are less than net subscriptions.
- Default time is a negative function of management fees and *posted* returns on Assets Under Management (AUM).

Paper 4: Policy and Regulation Implications II

Main findings

New lessons for operational risk capital requirements

Overall, it is not clear what the impact of the Madoff fraud will have on how operational capital requirements are calculated.

- New beta for the asset management industry?
- Impact on Advanced Measurement Approach

Under current rules, the impact is potentially tremendous, and may need special considerations.

Paper 4: Policy and Regulation Implications III

Main findings

Implications of the Madoff case for regulators and the investment industry

- Rethinking due diligence processes
  - Operational due diligence vs. Quantitative due diligence $\Longrightarrow$ lack of quantitative expertise.
  - Initiatives to define a common analysis framework: AIMA, HFWG, etc.

- Future of the HF industry: Rethinking it!
  - 2003-2007: HF bubble (like the Internet bubble).
  - 2008-2009: *Annus horribilis* (liquidity, gates, Madoff, etc.).
  - "Retailization" of the industry.

  - Promote transparency, liquidity and standardization
    - Platform of managed accounts.
    - Replication products (carry trades, volatility selling, etc.).
  - Role of HFR is not yet clear (*in my opinion*)
    - good potential for regulation as risk/control tool, in particular for FoHF
    - TO BE DEVELOPED

Future Perspectives

- Problems of aggregation, especially in the light of nonlinearities
- Several financial engineering applications can be developed
    - Asset Allocation: emphasis on non replicable HFs as holders of "talent"
    - Hedging portfolio of a portfolio of Hedge funds
- Other applications for Bayesian Filters?

Part III

Appendix

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Replicating with Kalman Filter

Decomposition of the yearly performance

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Period	Traditional		Alternative		Total
	Alpha	Beta	Alpha	Beta	
1994	0.43	1.13	0.68	0.88	1.56
1995	6.99	13.56	7.00	13.55	21.50
1996	11.77	8.35	12.18	7.95	21.10
1997	7.21	8.94	-2.61	19.93	16.79
1998	-3.98	6.87	-4.44	7.39	2.62
1999	15.56	13.62	7.96	21.61	31.29
2000	3.03	1.90	3.63	1.31	4.98
2001	4.08	0.53	2.11	2.47	4.62
2002	4.39	-5.59	0.74	-2.18	-1.45
2003	2.99	16.08	3.96	15.00	19.55
2004	1.23	7.71	1.83	7.08	9.03
2005	2.30	6.84	1.44	7.74	9.30
2006	2.32	10.33	1.10	11.67	12.89
2007	5.30	4.43	3.35	6.39	9.96
2008	-5.96	-5.13	-4.90	-6.19	-10.78
1994-2008	3.80	5.92	2.22	7.55	9.94
1997-2008	3.14	5.46	1.14	7.55	8.77
2000-2008	2.20	4.02	1.48	4.75	6.30

Proof of the input-output relationship in a Tracking System

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Proof.
It is assumed that the input-output relations can be described by

$$\begin{pmatrix} \mathbf{e} \\ \mathbf{z} \end{pmatrix} = \mathbf{\Gamma} \begin{pmatrix} \boldsymbol{\psi} \\ \mathbf{u} \end{pmatrix}$$

with $\mathbf{\Gamma}$ a real and proper matrix which can be partitioned as

$$\mathbf{\Gamma} = \begin{pmatrix} \mathbf{\Gamma}_{e\psi} & \mathbf{\Gamma}_{eu} \\ \mathbf{\Gamma}_{z\psi} & \mathbf{\Gamma}_{zu} \end{pmatrix}$$

Recall also that $\mathbf{u} = K\mathbf{z}$. Then, one can write

$$\mathbf{e} = \mathbf{T}_{e\psi}\psi$$

with $\mathbf{T}_{e\psi}$ the transfer function matrix from ψ to $\mathbf{e}$.
We have

$$\begin{aligned} \mathbf{e} &= \mathbf{\Gamma}_{e\psi}\psi + \mathbf{\Gamma}_{eu}K\mathbf{z} \\ \text{and } \mathbf{z} &= \mathbf{\Gamma}_{z\psi}\psi + \mathbf{\Gamma}_{zu}K\mathbf{z} \end{aligned}$$

Continues on next slide ...

Proof of the input-output relationship in a Tracking System

Proof (Cont'd).

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Thus,

$$\begin{aligned}(I - \boldsymbol{\Gamma}_{zu} K)\, z &= \boldsymbol{\Gamma}_{z\psi}\, \psi \\ \mathbf{z} &= (I - \boldsymbol{\Gamma}_{zu} K)^{-1}\, \boldsymbol{\Gamma}_{z\psi}\, \psi\end{aligned}$$

Therefore,

$$\begin{aligned}\mathbf{e} &= \boldsymbol{\Gamma}_{e\psi}\, \psi + \boldsymbol{\Gamma}_{eu} K\, (I - \boldsymbol{\Gamma}_{zu} K)^{-1}\, \boldsymbol{\Gamma}_{z\psi}\, \psi \\ &= \left[\boldsymbol{\Gamma}_{e\psi} + \boldsymbol{\Gamma}_{eu} K\, (I - \boldsymbol{\Gamma}_{zu} K)^{-1}\, \boldsymbol{\Gamma}_{z\psi}\right] \psi\end{aligned}$$

$\square$

Statistics Description

- $\hat{\mu}_{1\mathrm{Y}}$ is the annualized performance;
- π_{AB} the proportion of the HFRI index performance explained by the clone;
- σ_{TE} is the yearly tracking error;
- ρ, τ and ρ_S are respectively the linear correlation, the Kendall tau and the Spearman rho between the monthly returns of the clone and the HFRI index;
- s is the sharpe ratio;
- γ_1 is the skewness;
- γ_2 is the excess kurtosis.