AACD models – Summary

Notations

- $x_i = t_i - t_{i-1}$ where t_i: arrival time of event *i*. x_i is called the duration.
- I_i denotes all information available at time t_i

Model

- $x_i = g(\psi_i)\varepsilon_i$ where ε_i is the standardized duration. $\{\varepsilon_i\}$ are identically distributed with pdf f such that f has a strictly positive support and $\mathrm{E}\varepsilon_i = 1$.
- $\psi_i = \mathrm{E}(x_i|I_{i-1})$ is the conditional duration. One assumption of the model is that ψ_i and ε_i are independent.
- The evolution equation of ψ_i is given by

$$f_\lambda(\psi_i) = \frac{\psi_i^\lambda - 1}{\lambda} = \omega_* + \sum_{l=1}^{r} \alpha_l^* \, \psi_{i-l}^\lambda \mathrm{Shock}(\varepsilon_{i-l}) + \sum_{k=1}^{q} \beta_k \left(\frac{\psi_{i-k}^\lambda - 1}{\lambda}\right),$$

  i.e.

$$\psi_i^\lambda = \omega + \sum_{l=1}^{r} \alpha_l \, \psi_{i-l}^\lambda \mathrm{Shock}(\varepsilon_{i-l}) + \sum_{k=1}^{q} \beta_k \psi_{i-k}^\lambda$$

  where $\mathrm{Shock}(\varepsilon) = [|\varepsilon - b| - c(\varepsilon - b)]^\nu$, and $\omega = \lambda\omega_* - \sum_{k=1}^{q} \beta_k + 1$ and for $l = 1..r$, $\alpha_l = \lambda\alpha_l^*$. b is called the shift parameter and c is the rotation parameter. ν is a shape parameter that controls the convexity of the shock impact curve. λ is the Box-Cox parameter. It defines the Box-Cox transformation done on the conditional duration.
- $g(\psi_i)$ depends only on the distribution of ε_i, i.e. on f
  - $f \equiv \mathrm{Exponential}(1)$: $g(\psi_i) = \psi_i$
  - $f \equiv \mathrm{Weibull}(1,\gamma)$: $g(\psi_i) = \psi_i \cdot \frac{1}{\Gamma\left(1+\frac{1}{\gamma}\right)}$
  - $f \equiv \mathrm{Generalized\ Gamma}(1,\gamma,\kappa)$: $g(\psi_i) = \psi_i \cdot \frac{\Gamma(\kappa)}{\Gamma\left(\kappa+\frac{1}{\gamma}\right)}$
  - $f \equiv \mathrm{Burr}(\mu,\kappa,\sigma^2)$ under the constraint $\mu = 1$: $g(\psi_i) = \psi_i \cdot \frac{(\sigma^2)^{1+\frac{1}{\kappa}}\Gamma\left(\frac{1}{\sigma^2}+1\right)}{\Gamma\left(1+\frac{1}{\kappa}\right).\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)}$